Oxford Cambridge and RSA

Monday 20 May 2019 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

* 7 7 7 1 2 9 7 4 1 2 *

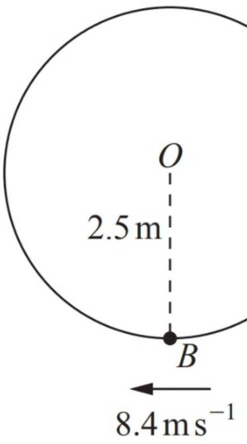
You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1

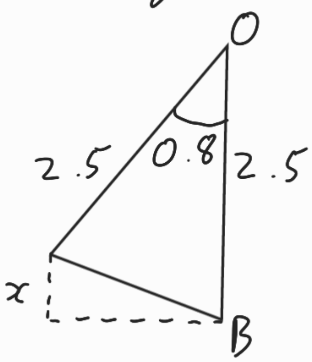


A smooth wire is shaped into a circle of radius 2.5 m which is fixed in a vertical plane with its centre at a point O. A small bead B is threaded onto the wire. B is held with OB vertical and is then projected horizontally with an initial speed of $8.4\,\mathrm{m\,s^{-1}}$ (see diagram).

(a) Find the speed of B at the instant when OB makes an angle of 0.8 radians with the downward vertical through O. **[3]**

Initial energy $= KE = \frac{1}{2} m \times 8.4^2$

Energy at 0.8 rad $= KE + GPE$



$x = 2.5 - 2.5\cos 0.8$

$\therefore E = \frac{1}{2} m v^2 + m \times 9.8 \times 2.5(1 - \cos 0.8)$

Energy is conserved $\Rightarrow E_{initial} = E_{0.8\,rad}$

$\frac{1}{2} \cancel{m} \times 8.4^2 = \frac{1}{2} \cancel{m} v^2 + \cancel{m} \times 9.8 \times 2.5(1 - \cos 0.8)$

$v^2 = 2 \times \left(\frac{1}{2} \times 8.4^2 - 9.8 \times 2.5(1 - \cos 0.8)\right)$

$v = \sqrt{55.698\ldots}$

$= 7.46\,\mathrm{ms^{-1}}$

(b) Determine whether B has sufficient energy to reach the point on the wire vertically above O. **[3]**

Energy required to reach top $= m \times 9.8 \times (2 \times 2.5)$
$= 49m$

Initial energy $= \frac{8.4^2}{2} m = 35.28m$

$49m > 35.28m$ $\therefore$ B does not have sufficient energy.

2 A particle A of mass 3.6 kg is attached by a light inextensible string to a particle B of mass 2.4 kg.

A and B are initially at rest, with the string slack, on a smooth horizontal surface. A is projected directly away from B with a speed of $7.2\,\mathrm{m\,s^{-1}}$.

(a) Calculate the speed of A after the string becomes taut. **[3]**

Conservation of momentum:

$3.6 \times (-7.2) + 2.4 \times 0 = 3.6 \times V_A + 2.4 \times V_B$

$3.6V_A + 2.4V_B = -25.92$ (taking right as +ve)

String is taut $\Rightarrow V_A = V_B$

$\therefore V_A = \frac{-25.92}{3.6 + 2.4} = -4.32\,\mathrm{m\,s^{-1}}$

(b) Find the impulse exerted on A at the instant that the string becomes taut. **[2]**

(c) Find the loss in kinetic energy as a result of the string becoming taut. **[2]**

b. I on $A = \Delta mv$ of A

$= 3.6 \times (-4.32) + 3.6 \times 7.2$

$= 10.4$ Ns (towards B)

(A and B effectively combine)

c. $\Delta KE = \left| \frac{1}{2} \times 3.6 \times 7.2^2 - \frac{1}{2} \times (3.6 + 2.4) \times 4.32^2 \right|$

$= 37.3$ J

3 A car of mass 1500 kg has an engine with maximum power 60 kW. When the car is travelling at $10\,\mathrm{m\,s^{-1}}$ along a straight horizontal road using maximum power, its acceleration is $3.3\,\mathrm{m\,s^{-2}}$.

In an initial model of the motion of the car it is assumed that the resistance to motion is constant.

(a) Using this initial model, find the greatest possible steady speed of the car along the road. **[4]**

Force $= \frac{\text{Power}}{\text{velocity}}$

Newton II: $\frac{60 \times 10^3}{10} - R = 1500 \times 3.3$

$R = 60 \times 10^2 - 1500 \times 3.3$

$R = 1050$ N

$\therefore$ at top speed: driving force = resistance

$\frac{60 \times 10^3}{v_{max}} = 1050$

$v_{max} = 57.1\ \mathrm{ms^{-1}}$

In a refined model the resistance to motion is assumed to be proportional to the speed of the car.

(b) Using this refined model, find the greatest possible steady speed of the car along the road. **[5]**

$R \propto v \Rightarrow R = kv$

Newton II: $\frac{60 \times 10^3}{10} - k \times 10 = 1500 \times 3.3$

$\therefore k = 105$

At new maximum speed: $\frac{60 \times 10^3}{v_{max}} = 105 \times v_{max}$

$v_{max} = \sqrt{571.4}$

$= 23.9 \text{ ms}^{-1}$

The greatest possible steady speed of the car on the road is measured and found to be $21.6\,\text{m}\,\text{s}^{-1}$.

(c) Explain what this value means about the models used in parts **(a)** and **(b)**. **[2]**

Constant resistance model in (a) is very inaccurate, 57.1 ms^{-1} is very different to 21.6 ms^{-1}.

The linear model in (b) gives a more accurate answer at this speed, but probably underestimates the resistance at higher speeds).

4 A student is studying the speed of sound, u, in a gas under different conditions.

He assumes that u depends on the pressure, p, of the gas, the density, ρ, of the gas and the wavelength, λ, of the sound in the relationship $u = kp^{\alpha}\rho^{\beta}\lambda^{\gamma}$, where k is a dimensionless constant. (The wavelength of a sound is the distance between successive peaks in the sound wave.)

(a) Use the fact that density is mass per unit volume to find $[\rho]$. **[1]**

(b) Given that the units of p are N m^{-2}, determine the values of α, β and γ. **[7]**

a. $[\rho] = ML^{-3}$

b. speed $= \dfrac{\text{distance}}{\text{time}} = LT^{-1}$

$F = ma$

pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{ML^{3}T^{-2}}{L^{2}} = MLT^{-2}$

density $= ML^{-3}$

wavelength $= L$

$\therefore\ u = kp^{\alpha}\rho^{\beta}\lambda^{\gamma} \Rightarrow LT^{-1} = M^{\alpha}L^{\alpha}T^{-2\alpha}M^{\beta}T^{-3\beta}L^{\gamma}$

Powers on LHS must equal powers on RHS:

M : $\alpha + \beta = 0$

T : $-2\alpha = -1$

$\underline{\alpha = \frac{1}{2}} \Rightarrow \underline{\beta = -\frac{1}{2}}$

L: $1 = -\alpha - 3\beta + \gamma$

$\gamma = 1 + \frac{1}{2} - \frac{3}{2}$

$\underline{\gamma = 0}$

(c) Comment on what the value of γ means about how fast sounds of different wavelengths travel through the gas. **[1]**

Sounds of all wavelengths have the same speed through the gas.

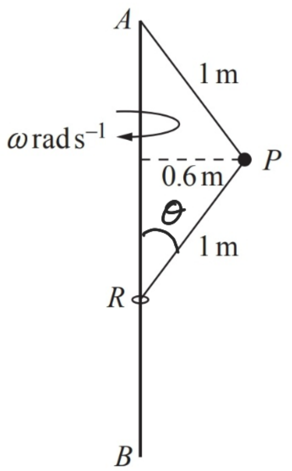
The student carries out two experiments, A and B, to measure u. Only the density of the gas varies between the experiments, all other conditions being unchanged. He finds that the value of u in experiment B is double the value in experiment A.

(d) By what factor has the density of the gas in experiment A been multiplied to give the density of the gas in experiment B? **[2]**

$$u \propto \rho^{-\frac{1}{2}} \Rightarrow 2 = \left(\frac{\rho_B}{\rho_A}\right)^{-\frac{1}{2}}$$

$$\frac{\rho_B}{\rho_A} = \frac{1}{2^2} = \frac{1}{4}$$

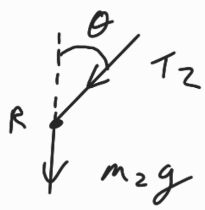
5



As shown in the diagram, AB is a long thin rod which is fixed vertically with A above B. One end of a light inextensible string of length 1 m is attached to A and the other end is attached to a particle P of mass m_1 kg. One end of another light inextensible string of length 1 m is also attached to P. Its other end is attached to a small smooth ring R, of mass m_2 kg, which is free to move on AB.

Initially, P moves in a horizontal circle of radius 0.6 m with constant angular velocity ω rad s^{-1}. The magnitude of the tension in string AP is denoted by T_1 N while that in string PR is denoted by T_2 N.

(a) By considering forces on R, express T_2 in terms of m_2. **[2]**



$$\theta = \sin^{-1}\left(\frac{0.6}{1}\right) = 0.644$$

Resolving vertically: $T_2 \cos 0.644 = m_2 g$

$$T_2 = \frac{5}{4} g m_2 \quad \text{or} \quad (T_2 = 12.25 m_2)$$

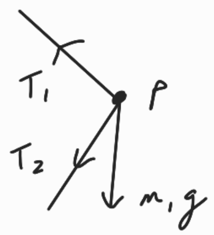
(both valid)

(b) Show that

(i) $T_1 = \frac{49}{4}(m_1 + m_2)$, **[2]**

(ii) $\omega^2 = \frac{49(m_1 + 2m_2)}{4m_1}$. **[3]**

i.



Resolving vertically:

$$T_2 \cos\theta + m_1 g = T_1 \cos\theta$$

$$T_1 = T_2 + \frac{m_1 g}{\cos\theta}$$

from (a)

$$T_1 = \frac{5}{4} g m_2 + \frac{5}{4} g m_1$$

$$\boxed{T_1 = \frac{49}{4}(m_1 + m_2)}$$ (as required)

ii. Resolve horizontally for P:

$$T_1 \sin\theta + T_2 \sin\theta = m_1 w^2 r$$ $(\cos\theta = 0.8 \Rightarrow \sin\theta = 0.6)$

from (i) from (a)

$$\frac{49}{4}(m_1 + m_2) \times 0.6 + \frac{49}{4} m_2 \times 0.6 = m_1 \times 0.6\, w^2$$

$$w^2 = \frac{7.35(m_1 + m_2) + 7.35 m_2}{0.6 m_1}$$

$$w^2 = \frac{7.35 m_1 + 14.7 m_2}{0.6 m_1}$$

$\left(\frac{7.35}{0.6} = \frac{49}{4}\right)$

$$\boxed{w^2 = \frac{49(m_1 + 2m_2)}{4m_1}}$$ (as required)

(c) Deduce that, in the case where m_1 is much bigger than m_2, $\omega \approx 3.5$. **[2]**

if $m_1 >> m_2$: $m_1 + 2m_2 \approx m_1$

$$\therefore w^2 \approx \frac{49m_1}{4m_1}$$

$$w \approx \sqrt{\frac{49}{4}} = 3.5$$

In a different case, where $m_1 = 2.5$ and $m_2 = 2.8$, P slows down. Eventually the system comes to rest with P and R hanging in equilibrium.

(d) Find the total energy lost by P and R as the angular velocity of P changes from the initial value of $\omega \text{ rad s}^{-1}$ to zero. **[5]**

$$\Rightarrow w = \sqrt{\frac{49 \times (2.5 + 2 \times 2.8)}{4 \times 2.5}}$$

$$= 6.3 \text{ rads}^{-1}$$

$$\therefore v = rw = 0.6 \times 6.3 = 3.78 \text{ ms}^{-1}$$

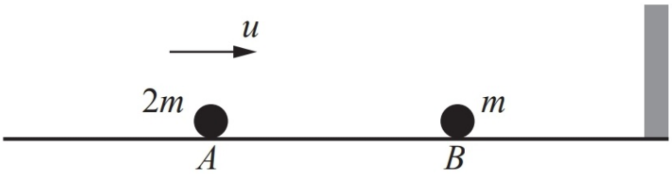
$\therefore$ initial energy $= \frac{1}{2} \times 2.5 \times 3.78^2 = 17.8605$ J

Finally: P drops from 0.8 m below A to 1 m below A
R drops from 1.6 m below A to 2 m below A

$\therefore \Delta PE$ for $P = 2.5 \times 9.8 \times (1 - 0.8) = 4.9$ J
ΔPE for $R = 2.8 \times 9.8 \times (2 - 1.6) = 10.976$ J

$\therefore$ Energy loss $= 17.8605 + 4.9 + 10.976$
$= 33.7$ J

6 Particles A of mass $2m$ and B of mass m are on a smooth horizontal floor. A is moving with speed u directly towards a vertical wall, and B is at rest between A and the wall (see diagram).



A collides directly with B. The coefficient of restitution in this collision is $\frac{1}{2}$.

B then collides with the wall, rebounds, and collides with A for a second time.

(a) Show that the speed of B after its second collision with A is $\frac{1}{2}u$. **[6]**

1st collision between A and B:

Conservation of momentum:

$2mu + 0m = 2mv_A + mv_B$

$2u = 2v_A + v_B$

$v_B = 2u - 2v_A$ ①

Newton's law of restitution:

$\frac{1}{2} = \frac{v_B - v_A}{u}$ ②

Sub ① into ②: $\frac{1}{2} = \frac{2u - 2v_A - v_A}{u}$

$3v_A = \frac{3}{2}u$

$v_A = \frac{1}{2}u$

2nd collision between A and B:

COM:

$2m \times \frac{1}{2}u + mU_B = 2mV_A + mV_B$

$u + U_B = 2V_A + V_B$

①

NLR:

$\frac{1}{2} = \frac{V_B - V_A}{\frac{1}{2}u - U_B}$

$u - 2U_B = 4V_B - 4V_A$

②

$2\times① + ② \quad : \quad 2u + \cancel{2U_B} + u - \cancel{2U_B} = \cancel{4V_A} + 2V_B + 4V_B \cancel{- 4V_A}$

$$3u = 6V_B$$

$$\Rightarrow V_B = \frac{1}{2}u$$

The first collision between A and B occurs at a distance d from the wall. The second collision between A and B occurs at a distance $\frac{1}{5}d$ from the wall.

(b) Find the coefficient of restitution for the collision between B and the wall. **[5]**

from (a) : $2u = 2v_A + v_B$ and $v_A = \frac{1}{2}u$

$$\Rightarrow 2u = u + v_B$$

$$\therefore v_B = u$$

1st collision between B and wall : $e = \pm \frac{U_B}{u}$

$$U_B = \pm eu$$

Time between ball collisions :

for A $= \frac{\text{distance}}{\text{speed}} = \frac{\frac{4}{5}d}{\frac{1}{2}u} = \frac{8d}{5u}$

for B $= \frac{\text{distance to wall}}{\text{speed to wall}} + \frac{\text{distance from wall}}{\text{speed from wall}} = \frac{d}{u} + \frac{\frac{1}{5}d}{eu}$

$$\Rightarrow \frac{8d}{5u} = \frac{d}{u} + \frac{d}{5eu}$$

$$\frac{8}{5} = 1 + \frac{1}{5e}$$

$$\frac{3}{5} = \frac{1}{5e}$$

$$e = \frac{1}{3}$$